# **Magnetic properties and M6ssbauer spectra of gamma ferric oxide and doped gamma ferric oxide**

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Magnetic hysteresis, M6ssbauer spectra and temperature variation of initial magnetic susceptibility of thirteen samples of doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> containing cobalt or gadolinium are determined. The samples containing more than 1.0% cobalt are found to have a multi-domain configuration, and undoped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>, gadolinium-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> and doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> containing less than 1.0% (except 0.3%) cobalt have a single domain configuration. M6ssbauer spectra of gadoliniumdoped samples suggest that gadolinium occupies A and B sites. In cobalt-doped samples, the effective magnetic fields at A and B sites are different at room temperature and liquid nitrogen temperature. The samples which have a multi-domain configuration display an additional central doublet in M6ssbauer spectra indicating that these samples contain multi-domain clusters. The saturation magnetization of gadolinium-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> is much lower, and the coercive force of cobalt-doped samples is much higher than of gadolinium-doped and undoped samples.

## **1. Introduction**

Gamma ferric oxide ( $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>) is a ferrimagnetic compound with the spinel structure, in which iron(III) cations occupy both the tetrahedral (A) and octahedral (B) sites, and is usually assumed to have a collinear magnetic structure consisting of two sublattices. The measurements of direct current electrical conductivity of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>, cobalt-, nickel-, gadolinium- and terbiumdoped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> prepared in static air [1, 2] carried out at different temperatures suggest the presence of adsorbed oxygen and water in  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>. These measurements determined in the different atmospheres, supplemented with the data of infrared spectra and X-ray diffraction patterns, indicate the presence of lattice water and of hydrogen ferrite phase. An addition of a small amount of cobalt to  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> results in a marked change of the magnetic properties of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> [3–6]. It was, therefore, of interest to study the effect of divalent as well as trivalent dopants on the magnetic properties of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>. In the present work, doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> samples with different dopants have been synthesized under controlled experimental conditions.

## **2. Experimental procedure**

The detailed synthesis of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>, cobalt- and gadolinium-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> by dehydration of  $\gamma$ -FeOOH are reported earlier [2]. The procedures used for the measurement of X-ray diffraction patterns and magnetic properties are also given earlier [1, 7]. The constant acceleration M6ssbauer spectrometer assembled in the Department of Physics, University of Poona, was used in the present work. The Mössbauer spectra were taken at room temperature and liquid nitrogen temperature. The hyperfine interaction parameters were computed using an interactive least squares "MOSFIT" Program adopted to 1CL 1904S computer. The measurements of temperature variation of initial magnetic susceptibility were determined by a double-coil method [8, 9] assembled at the Indian Institute of Technology, Bombay, using a field of 1.00e, and a frequency of 263 Hz. The temperature was varied between 20 and  $550^{\circ}$ C, and was measured accurately within  $\pm 1^{\circ}$  C.

#### **3. Results and discussion**

At room temperature, the experimental value of saturation magnetization of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> obtained from ferrous oxalate dihydrate (sample A) is  $7.10 \text{ T kg}^{-1}$ , and that obtained from gamma ferric oxide hydroxide (sample B) is  $6.15 \text{ T kg}^{-1}$ . We may therefore assume that sample B is mainly hydrogen ferrite. When sample B is heated in a nitrogen atmosphere at  $280^{\circ}$  C for 12 h and is cooled in the same atmosphere (sample C), its saturation magnetization is enhanced to  $7.15 \text{ T kg}^{-1}$ . There is no change in the saturation magnetization when a similar heat treatment is given in static air. This observation suggests that hydrogen ferrite is transformed to  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> after heating for 12 h in nitrogen. Thus sample B can be assumed to be the hydrogendoped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>. Because the saturation magnetization of sample A is somewhat lower than that expected for the vacancy-ordered  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>, it may be considered to be a mixture of vacancy-ordered  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> and hydrogen ferrite.

M6ssbauer spectra of samples A, B and C consist of six well-resolved narrow bands (half-band width  $0.285$  mm sec<sup>-1</sup>) (Fig. 1) in the intensity ratio 3:2:1:



*Figure 1* Computer-fitted room-temperature Mössbauer spectra of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>.

1:2:3. The best fit of the experimental curve is obtained as superposition of two six bands (see Fig. 1). The line width corresponding to the B-site  $(0.27 +$  $0.03$  mm sec<sup>-1</sup>) is found to be slightly larger than that for the A-site  $(0.22 \pm 0.03 \text{ mm sec}^{-1})$ . The isomer shift and hyperfine field of these samples are very comparable (Table I). No quadrupole splitting is observed for both the sites. The area ratio of the B-site to the A-site sextet was found to be 1.62. Therefore, the area ratio of these samples should, theoretically, lie between 1.50 and 1.67 depending upon the percentage of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> with hydrogen and vacancy, respectively [10]. Because sample B can be considered to be hydrogen-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>, it will be profitable to synthesize the doped samples of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> using the 7-FeOOH method.

The X-ray diffraction patterns of cobalt- and gadolinium-doped samples are similar to that of the undoped sample of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>, and corresponding to a tetragonal configuration [11]. From X-ray diffraction it is observed that the intensity of the (3 2 1) line is greater in the cobalt-doped samples and is less in the case of gadolinium-doped samples. The presence of the (3 2 1) line indicates an increase in the density of atoms in the  $(321)$  plane. This is possible through replacement of hydrogen and/or occupation of vacant

Sample no.	Concentration of dopants	Coercive force, $H_c$ $(\pm 0.25 \text{ A m}^{-1})$	Saturation magnetization, $M_s$ $(\pm 0.04 \text{ T kg}^{-1})$	Ratio of remanence coercive force to coercive force $(H_r/H_c)$	Ratio of $M_{\rm R}/M_{\rm s}$	Hyperfine field, $H_n$ ( $\pm$ 5.0 KOe)	
						A-site	<b>B</b> -site
A	$\gamma$ -Fe <sub>2</sub> O <sub>3</sub> from $Fe2O4 \cdot 2H2O$	3.52	7.10	2.00	0.64	493.8	496.1
$\bf{B}$	$\gamma$ -Fe <sub>2</sub> O <sub>3</sub> from $\gamma$ -FeOOH	3.52	6.15	2.00	0.65	493.8	496.0
$\mathbf C$	Sample B heated at $280^{\circ}$ C in N <sub>2</sub> atmosphere	3.52	7.15	2.00	0.66	494.1	496.3
I	Co doped: 0.3% 0.5% 0.75% $1.0\%$ $3.0\%$	3.52	6.07	2.03	0.63	488.6	494.9
$\mathbf{I}$		3.58	5.88	2.00	0.61	483.0	490.0
Ш		3.69	5.86	2.03	0.63	482.0	488.0
IV		3.87	5.68	2.11	0.56	478.8	485.0
V		5.28	5.60	2.14	0.58	476.3	483.0
VI	$5.0\%$	5.28	5.56	2.28	0.57	473.4	481.8
VII	$10.0\%$	6.03	5.56	2.25	0.59	470.9	480.0
VIII	15.0%	6.54	5.20	2.31	0.58	470.0	479.1
IX	Gd doped: 0.3%	3.49	5.02	1.84	0.58	455.0	458.5
X	0.50%	3.52	5.06	1.84	0.57	456.1	458.5
XI	0.75%	3.46	5.00	1.82	0.60	455.8	458.5
XII	$1.0\%$	3.46	4.60	1.83	0.58	456.0	458.5
XIII	1.5%	3.52	3.80	1.82	0.59	455.6	458.4

TABLE I Magnetic properties and Mössbauer spectra\* of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> and doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> at room temperature

\*The computer-fitted Mössbauer spectra show negligible quadrupole splitting, and the isomer shift (for A site  $0.24 \text{ mm sec}^{-1}$  and for B site  $0.32$  mm sec<sup>-1</sup> with respect to iron foil) remains constant within experimental accuracy.



site by cobalt and gadolinium, specifically on the B-site. This is a significant new observation which directly gives evidence of preferential diffusion of cobalt or gadolinium in the  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> lattice during the precipitation process.

The observed saturation magnetization values of cobalt- and gadolinium-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> samples are lower than those reported in literature [12, 13]. A direct proportionality between the hyperfine field  $(H_n)$ and saturation magnetization  $(M<sub>s</sub>)$  for the magnetically similar compounds has been suggested by Marshell [14, 15]. It is interesting to note that the separate Marshell straight lines (Fig. 2) are obtained for cobaltand gadolinium-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>. Thus the magnetic configuration of these doped samples is not identical, although the respective group of doped samples has similar magnetic configuration.

For such doped samples containing a low concentration of gadolinium, we do not expect the formation of a garnet, and the X-ray diffraction patterns of these samples resemble the pattern of undoped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>. The gadolinium(III) ion with  $4f<sup>7</sup>$  electronic configuration is spherically symmetrical, and the crystal field effects are comparatively negligible. An abnormally low value of observed saturation magnetization (Table I), even for a sample containing a low concentration of gadolinium, suggests that part of the gadolinium also occupies A-sites. Because the exchange interactions between gadolinium(III) and the neigh-

bouring iron(III) ions are usually about an order of magnitude smaller than those found between two neighbouring iron(III) ions [16], fluctuations of the magnetization direction about the energy minimum are envisaged. Such fluctuations will lead to a decrease in the hyperfine field. The M6ssbauer spectra of gadolinium(III)-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> are according to this prediction (Fig. 3). These spectra are similar to that of undoped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> and consist of six symmetrical narrow bands except that the hyperfine field is abnormally low (Table I). The isomer shifts of these samples are slightly different from undoped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>.

Room-temperature M6ssbauer spectra of all the cobalt-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> samples displayed a broadened six-finger pattern along with the central adsorption (Figs 4 and 5). The broadening of the M6ssbauer line is probably associated with the contribution of the following configuration.

$$
Fe^{3+}_{8}[(Fe^{2+}_{x}M^{2+}_{y}H^{+}_{4-2x-2y}\square_{x-y})Fe^{3+}_{12}]O_{32}
$$

In addition to the broadening of the Mössbauer lines, doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> containing 0.3% and more than 1.0% cobalt show a well-resolved doublet in the centre of the M6ssbauer spectra. The M6ssbauer parameters for the sextets are given in Table I.

The isomer shift of cobalt-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> for the A-site almost remains the same as that of undoped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>. For the B-site of cobalt-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> an increase in isomer shift (0.59  $\pm$  0.03 mm sec<sup>-1</sup>) value



Figure 3 Room-temperature Mössbauer spectra of gadoliniumdoped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>.

compared to the undoped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> is seen. This probably shows that although there is not much change in the bonding of iron atoms at the A-site, some change in the bonding does take place at the octahedral B-site due to cobalt concentration. The hyperfine field for all the cobalt-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> samples are found to be less than that of pure  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> (see Table I). The hyperfine field for both A and B sites is found to be different. This clearly indicates that the addition of cobalt has created nonequivalent sites of iron ions on the B-site. The area ratio of the B-site sextet to the A-site sextet was found to be the same. Therefore, cobalt enters vacant sites or replaces iron atoms or hydrogen.

The two doublets representing the central absorption show an interesting behaviour. The isomer shift of one doublet remained almost the same within about  $0.11$  mm sec<sup>-1</sup> for all the samples. For the other doublet, the isomer shift remained almost the same at  $0.51$  mm sec<sup>-1</sup> for all the samples. These isomer shift (IS) values clearly indicate that the valence state of iron ions responsible for producing the doublets is  $Fe<sup>3+</sup>$ . The quadrupole splitting of the first doublet (with low IS) ranges between 0.62 and 0.93 mm sec<sup>-1</sup>  $(\pm 0.03$  mm sec<sup>-1</sup>) and that for the other doublet (with high IS) ranges between 0.77 and  $1.02 \text{ mm sec}^{-1}$  $(\pm 0.03 \text{ mm sec}^{-1})$ . The low value of the quadrupole splitting also indicates that the doublets must be due to  $Fe<sup>3+</sup>$  ions. Scanning electron microscopy and par-





Figure 4 Room-temperature Mössbauer spectra of 0.3 to 3.0%

ticle size distribution of doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> containing  $0.3\%$  and more than 1.00% cobalt show a wide particle size distribution ranging from 80 nm to about 140 nm, while the other doped sample and undoped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> have a particle size of 105 to 110 nm.

When a sample contains particles of various sizes, it will be characterized by a range of spin-flip time,  $\tau_F$ , which is given by  $[17]$ 

$$
\tau_{\rm F} = \tau_0 \exp (K V_{\rm eff}/K_{\rm B}T)
$$

where K is the anisotropy,  $V_{\text{eff}}$  the volume of the particles,  $K_B$  the Boltzmann constant, T the temperature,  $\tau_F$  the relaxation time of spin flip and  $\tau_0$  lies between  $10^{-10}$  and  $10^{-12}$  sec. For static cases,  $\tau_F \gg \tau_L$ , where  $\tau_L$  is the Larmor precession time and under such conditions a six-finger pattern is observed. For  $\tau_F \ll \tau_L$  superparamagnetism is observed. The existence of a spread in volume,  $V_{\text{eff}}$ , and the extremely strong dependence of  $\tau_F$  upon  $V_{\text{eff}}$  ensures that the range of  $\tau_F$  encompasses both extremes. It is, therefore, natural to expect the coexistence of a central peak and hyperfine structure. It is assumed that doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> containing 0.3% and more than 1.00% cobalt is not perfectly homogeneous but is comprised of a set of independent magnetic regions. We may then



Figure 5 Room-temperature Mössbauer spectra of 5.0 to 15.0% cobalt-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>.

imagine each such volume,  $V_{\text{eff}}$ , flipping back and forth with a flip time,  $\tau_F$ . At low temperature, the energy barrier  $KV_{\text{eff}}$  is high and thus  $\tau_F$  is long compared to  $\tau_L$ . Hence no relaxation effects should be observed. As temperature approaches  $T_N$  (Neel temperature),  $K \rightarrow 0$  and the spin-flip frequency increases, so that relaxation phenomena corresponding to superparamagnetism may be discerned. Because there is a spread in  $KV_{\text{eff}}$ , it gives rise to the simultaneous presence of a strong central absorption and hyperfine structure in the Mössbauer spectrum. In view of the scanning electron microscopy, the above argument seems to be more probable.

Mössbauer spectra of 3.0% cobalt-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> are recorded at liquid nitrogen temperature and also at 373 K. It is observed that the central absorption disappeared completely at liquid nitrogen temperature but, on the other hand, it increased considerably at temperatures higher than room temperature, *i.e.*  $373 K$  (Fig. 6). These observations indicate that the central absorption is due to the superparamagnetic component present in  $0.3\%$  and more than  $1.0\%$ cobalt-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>. Therefore, the two doublets could be considered to be superparamagnetic spectra displayed by  $Fe<sup>3+</sup>$  ion on tetrahedral and octahedral sites, respectively. The quadrupole interaction experienced by the superparamagnetic  $Fe^{3+}$  ions on the octahedral site and the concentration of cobalt ions could possibly distort the symmetry on the tetrahedral site giving rise to the quadrupole interaction experienced by  $Fe<sup>3+</sup>$  ions on the tetrahedral site. The initial magnetic susceptibility measurement of these samples will help to analyse the presence of the superparamagnetic particles and the clusters of particles with single domains.

The study of the temperature variation of initial magnetic susceptibility  $(X_i)$  curves of samples A, B and C show a typical Hopkinson effect [18]. For these samples,  $X_i$  increases with increasing temperature giving a peak value at the transformation temperature (Fig. 7b). This is a characteristic behaviour of samples having a single domain structure. Similarly, samples II to IV and IX to XIII are found to be single domain from the  $X_i$ -T behaviour.

The  $X_i$  values of samples I and V to VIII do not show any observable change in the temperature range 25 to 405°C. A sharp decrease is observed at the Curie temperature (405 $^{\circ}$ C) which corresponds to an irreversible transformation of  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> and  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> (Fig. 7a). These samples thus display the  $X_i-T$  behaviour characteristic of a multidomain structure [18]. The  $X_i$ -T curve of superparamagnetic samples shows  $X_i$  to decrease as  $T$  increases.

On the basis of these criteria, the samples which do not show the central Mössbauer absorption are found to be of single domain configuration, and the samples which show the central Mössbauer absorption have multidomain configuration. Because the central Mössbauer absorption is supposed to be due to the paramagnetic species, the samples which show the  $X_i$ -T behaviour characteristic of multidomain grains, are probably clusters of particles having single domains. It is significant to note here that the particle size of samples used in the present work was 80 to 140 nm.

All the samples studied in the present work showed a definite hysteresis loop at room temperature and liquid nitrogen temperature. The coercive force of cobalt-doped samples was much higher than that of other samples (Table I). It is interesting to observe that the coercive force  $(H_c)$ , saturation magnetization  $(M<sub>s</sub>)$  and  $M<sub>R</sub>/M<sub>s</sub>$  are increased for samples I, V to VII at liquid nitrogen temperature. The plot of  $H_c M_s$ against cobalt-doped concentration (figures not shown), does not show a linear trend at room temperature, but does show a reasonable linear relationship at liquid nitrogen temperature. It is already observed from the  $X_i$ -T curve that most of the cobalt-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> sample contains single domain particles and because  $H_{\rm c}M_{\rm s}$  is proportional to anisotropy in the case of single domain particles, one can conclude that the anisotropy is proportional to cobalt concentration at liquid nitrogen temperature. The deviation from the linear relationship at room temperature could be due to the clusters of single domains present in samples I, V to VII. The ratio of the remanence coercive force to coercive force of cobalt-doped samples is higher than that of the undoped sample; this ratio does not show any significant change on doping with gadolinium



(Table I). Thus there is an enhancement of magnetic anisotropy on cobalt doping.

#### 4. Conclusions

1. The low value of hyperfine field and the saturation magnetization of gadolinium-doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> suggest that the gadolinium ion occupies both A and B sites and the magnetization direction of gadolinium ion fluctuates about the energy minimum.

2. Doped  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> containing 0.3% and more than  $1.0\%$  cobalt shows a well-resolved doublet in the centre of the six-line Mössbauer spectra indicating the presence of some type of paramagnetic species in these samples. The data on the temperature variation of the



Figure 7 Plot of  $X_i/X_{i,\pi}$  against T for (a) multidomain structure, (b) single domain structure. ( $\bullet$ ,  $\circ$ ) Heating cycle, ( $\times$ ) cooling cycle.

**initial magnetic susceptibility suggest that these samples have a multidomain configuration. Thus the central doublet is associated with single domain clusters. All other samples are found to be single domain.** 

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